# Package 'gsl'

January 24, 2023

**Version** 2.1-8

**Depends** R (>= 4.0.0)

**Title** Wrapper for the Gnu Scientific Library

**SystemRequirements** Gnu Scientific Library version >= 2.5

**Description** An R wrapper for some of the functionality of the
Gnu Scientific Library.

**Maintainer** Robin K. S. Hankin <hankin.robin@gmail.com>

**License** GPL-3

**URL** https://github.com/RobinHankin/gsl

**BugReports** https://github.com/RobinHankin/gsl/issues

**NeedsCompilation** yes

**Author** Robin K. S. Hankin [aut, cre] (<https://orcid.org/0000-0001-5982-0415>),
Andrew Clausen [ctb] (multimin functionality),
Duncan Murdoch [ctb] (qrng functions)

**Repository** CRAN

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## R topics documented:

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`gsl-package` *Wrappers for the Gnu Scientific Library*

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## Description

An R wrapper for some of the functionality of the Gnu Scientific Library.

## Details

The DESCRIPTION file:

| | |
|---|---|
| Package: | gsl |
| Version: | 2.1-8 |
| Depends: | R (>= 4.0.0) |
| Title: | Wrapper for the Gnu Scientific Library |
| Authors@R: | c(person(given=c("Robin", "K. S."), family="Hankin", role = c("aut","cre"), email="hankin.robin@g |
| SystemRequirements: | Gnu Scientific Library version >= 2.5 |
| Description: | An R wrapper for some of the functionality of the Gnu Scientific Library. |
| Maintainer: | Robin K. S. Hankin <hankin.robin@gmail.com> |
| License: | GPL-3 |
| URL: | https://github.com/RobinHankin/gsl |
| BugReports: | https://github.com/RobinHankin/gsl/issues |
| NeedsCompilation: | yes |
| Author: | Robin K. S. Hankin [aut, cre] (<https://orcid.org/0000-0001-5982-0415>), Andrew Clausen [ctb] (mu |

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Index of help topics:



The function naming scheme directly copies the GSL manual except that leading `gsl_sf_` and, if present, the trailing `_e` is stripped: thus `gsl_sf_Airy_Ai_e` goes to R function `airy_Ai()`; however, some functions retain the prefix to avoid conflicts (viz `gsl_sf_sin()`, `gsl_sf_cos()`, `gsl_sf_gamma()`, `gsl_sf_ choose()`, `gsl_sf_beta()`).

R function arguments have the same names as in the GSL reference manual, except for the quasi-random functions documented in the `Qrng` manpage.

The package is organized into units corresponding to GSL header files; the `.c`, `.R`, and `.Rd` filenames match the GSL header filenames, except that the `.Rd` files are capitalized. Functions appear in all

files in the same order as the GSL reference manual, which precludes the use of the tidying method given in section 3.1 of R-exts. Error forms of GSL functions (_e versions) are used if available.

In general, documentation is limited to: (a), a pointer to the GSL reference book, which would in any case dominate any docs here; and (b), re-productions of some tables and figures in Abramowitz and Stegun (June 1964).

## Author(s)

Robin K. S. Hankin [aut, cre] (<https://orcid.org/0000-0001-5982-0415>), Andrew Clausen [ctb] (multimin functionality), Duncan Murdoch [ctb] (qrng functions)

Maintainer: Robin K. S. Hankin <hankin.robin@gmail.com>

## References


- M. Abramowitz and I. A. Stegun 1965. *Handbook of mathematical functions*. New York: Dover
- M. Galassi et al. 2007. *GNU Scientific Library*. Reference Manual edition 1.10, for GSL version 1.10; 10 September 2007
- R. K. S. Hankin 2006. *Introducing gsl, a wrapper for the Gnu Scientific Library*. Rnews 6(4):24-26


## Examples

```
airy_Ai(1:5)
```

---

`Airy` *Airy functions*

---

## Description

Airy functions as per the Gnu Scientific Library, reference manual section 7.4 and AMS-55, section 10.4. These functions are declared in header file `gsl_sf_airy.h`

## Usage

```
airy_Ai(x, mode=0, give=FALSE, strict=TRUE)
airy_Ai_scaled(x, mode=0, give=FALSE, strict=TRUE)
airy_Ai(x, mode=0, give=FALSE, strict=TRUE)
airy_Bi_scaled(x, mode=0, give=FALSE, strict=TRUE)
airy_Ai_deriv(x, mode=0, give=FALSE, strict=TRUE)
airy_Bi_deriv(x, mode=0, give=FALSE, strict=TRUE)
airy_Ai_deriv_scaled(x, mode=0, give=FALSE, strict=TRUE)
airy_Bi_deriv_scaled(x, mode=0, give=FALSE, strict=TRUE)
airy_zero_Ai(n, give=FALSE, strict=TRUE)
airy_zero_Bi(n, give=FALSE, strict=TRUE)
airy_zero_Ai_deriv(n, give=FALSE, strict=TRUE)
airy_zero_Bi_deriv(n, give=FALSE, strict=TRUE)
```

## Arguments

| | |
|---|---|
| x | input: real values |
| n | input: integer values |
| give | Boolean with TRUE meaning to return a list of three items: the value, an estimate of the error, and a status number |
| mode | input: mode. For GSL_PREC_DOUBLE, GSL_PREC_SINGLE, GSL_PREC_APPROX use 0, 1, 2 respectively |
| strict | Boolean, with TRUE meaning to return NaN if status is an error |

## Details

The zero functions return a status of GSL_EDOM and a value of NA for $n \leq 0$.

An example is given in the package vignette.

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
x <- seq(from=0,to=1,by=0.01)

f <- function(x){
cbind(x=x, Ai= airy_Ai(x), Aidash= airy_Ai_deriv(x),
Bi=airy_Ai(x),Bidash=airy_Bi_deriv(x))
}

f(x)  #table 10.11, p475
f(-x) #table 10.11, p476


x <- 1:10  #table 10.13, p478
cbind(x,
 airy_zero_Ai(x), airy_Ai_deriv(airy_zero_Ai(x)),
 airy_zero_Ai_deriv(x), airy_Ai(airy_zero_Ai_deriv(x)),
 airy_zero_Bi(x), airy_Bi_deriv(airy_zero_Bi(x)),

 airy_zero_Bi_deriv(x), airy_Bi(airy_zero_Bi_deriv(x))
 )


# Verify 10.4.4 and 10.4.5, p446:
3^(-2/3)/gamma(2/3)   - airy_Ai(0)
3^(-1/3) / gamma(1/3) + airy_Ai_deriv(0)
```

```
3^(-1/6) / gamma(2/3) - airy_Bi(0)
3^(1/6) / gamma(1/3)  - airy_Bi_deriv(0)
#  All should be small
```

---

`Bessel` *Bessel functions*

---

## Description

Bessel functions as per the Gnu Scientific Library, reference manual section 7.5 and AMS-55, chapters 9 and 10. These functions are declared in header file `gsl_sf_bessel.h`

## Usage

```
bessel_J0(x, give=FALSE, strict=TRUE)
bessel_J1(x, give=FALSE, strict=TRUE)
bessel_Jn(n,x, give=FALSE, strict=TRUE)
bessel_Jn_array(nmin,nmax,x, give=FALSE, strict=TRUE)
bessel_Y0(x, give=FALSE, strict=TRUE)
bessel_Y1(x, give=FALSE, strict=TRUE)
bessel_Yn(n,x, give=FALSE, strict=TRUE)
bessel_Yn_array(nmin, nmax, x, give=FALSE, strict=TRUE)
bessel_I0(x, give=FALSE, strict=TRUE)
bessel_I1(x, give=FALSE, strict=TRUE)
bessel_In(n, x, give=FALSE, strict=TRUE)
bessel_In_array(nmin, nmax, x, give=FALSE, strict=TRUE)
bessel_I0_scaled(x, give=FALSE, strict=TRUE)
bessel_I1_scaled(x, give=FALSE, strict=TRUE)
bessel_In_scaled(n, x, give=FALSE, strict=TRUE)
bessel_In_scaled_array(nmin, nmax, x, give=FALSE, strict=TRUE)
bessel_K0(x, give=FALSE, strict=TRUE)
bessel_K1(x, give=FALSE, strict=TRUE)
bessel_Kn(n, x, give=FALSE, strict=TRUE)
bessel_Kn_array(nmin, nmax, x, give=FALSE, strict=TRUE)
bessel_K0_scaled(x, give=FALSE, strict=TRUE)
bessel_K1_scaled(x, give=FALSE, strict=TRUE)
bessel_Kn_scaled(n, x, give=FALSE, strict=TRUE)
bessel_Kn_scaled_array(nmin, nmax, x, give=FALSE, strict=TRUE)
bessel_j0(x, give=FALSE, strict=TRUE)
bessel_j1(x, give=FALSE, strict=TRUE)
bessel_j2(x, give=FALSE, strict=TRUE)
bessel_jl(l,x, give=FALSE, strict=TRUE)
bessel_jl_array(lmax,x, give=FALSE, strict=TRUE)
bessel_jl_steed_array(lmax, x, give=FALSE, strict=TRUE)
bessel_y0(x, give=FALSE, strict=TRUE)
```

```
bessel_y1(x, give=FALSE, strict=TRUE)
bessel_y2(x, give=FALSE, strict=TRUE)
bessel_yl(l, x, give=FALSE, strict=TRUE)
bessel_yl_array(lmax, x, give=FALSE, strict=TRUE)
bessel_i0_scaled(x, give=FALSE, strict=TRUE)
bessel_i1_scaled(x, give=FALSE, strict=TRUE)
bessel_i2_scaled(x, give=FALSE, strict=TRUE)
bessel_il_scaled(l, x, give=FALSE, strict=TRUE)
bessel_il_scaled_array(lmax, x, give=FALSE, strict=TRUE)
bessel_k0_scaled(x, give=FALSE, strict=TRUE)
bessel_k1_scaled(x, give=FALSE, strict=TRUE)
bessel_k2_scaled(x, give=FALSE, strict=TRUE)
bessel_kl_scaled(l,x, give=FALSE, strict=TRUE)
bessel_kl_scaled_array(lmax,x, give=FALSE, strict=TRUE)
bessel_Jnu(nu, x, give=FALSE, strict=TRUE)
bessel_sequence_Jnu(nu, v, mode=0, give=FALSE, strict=TRUE)
bessel_Ynu(nu, x, give=FALSE, strict=TRUE)
bessel_Inu(nu, x, give=FALSE, strict=TRUE)
bessel_Inu_scaled(nu, x, give=FALSE, strict=TRUE)
bessel_Knu(nu, x, give=FALSE, strict=TRUE)
bessel_lnKnu(nu, x, give=FALSE, strict=TRUE)
bessel_Knu_scaled(nu, x, give=FALSE, strict=TRUE)
bessel_zero_J0(s, give=FALSE, strict=TRUE)
bessel_zero_J1(s, give=FALSE, strict=TRUE)
bessel_zero_Jnu(nu, s, give=FALSE, strict=TRUE)
```

## Arguments

| | |
|---|---|
| `x,v,nu` | input: real valued |
| `n,nmin,nmax,lmax` | input: integer valued |
| `l,s` | input: integer valued |
| `mode` | Integer, calc mode |
| `give` | Boolean with `TRUE` meaning to return a list of three items: the value, an estimate of the error, and a status number |
| `strict` | strict or not |

## Details

All as for the GSL reference manual section 7.5

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
# Compare native R routine with GSL:
besselK(0.55,4) - bessel_Knu(4,0.55)  # should be small


x <- seq(from=0,to=15,len=1000)
plot(x,bessel_J0(x),xlim=c(0,16),ylim=c(-0.8,1.1),type="l",
           xaxt="n",yaxt="n",bty="n",xlab="",ylab="",
           main="Figure 9.1, p359")
jj.Y0 <- bessel_Y0(x)
jj.Y0[jj.Y0< -0.8] <- NA
lines(x,jj.Y0)
lines(x,bessel_J1(x),lty=2)
jj.Y1 <- bessel_Y1(x)
jj.Y1[jj.Y1< -0.8] <- NA
lines(x,jj.Y1,lty=2)
axis(1,pos=0,at=1:15,
     labels=c("","2","","4","","6","","8","","10","","12","","14","") )
axis(2,pos=0,at=seq(from=-8,to=10,by=2)/10,
labels=c("-.8","-.6","-.4","-.2","0",".2",".4",".6",".8","1.0"))
arrows(0,0,16,0,length=0.1,angle=10)
arrows(0,0,0,1.1,length=0.1,angle=10)
text(1.1, 0.83, expression(J[0]))
text(0.37, 0.3, expression(J[1]))
text(0.34,-0.3, expression(Y[0]))
text(1.7,-0.5, expression(Y[1]))
text(4.2, 0.43, expression(Y[1]))
text(7.2, 0.33, expression(J[0]))
text(8.6, 0.3, expression(J[0],paste("    ,")))
text(9.1, 0.3, expression(Y[0]))

x <- seq(from=0,to=13,len=100)
y <- t(bessel_jl_array(3,x))
y[y>0.6] <- NA
 matplot(x,y,col="black",type="l",xaxt="n",yaxt="n",bty="n",
         xlab="",ylab="",xlim=c(0,16),ylim=c(-0.3,0.75),
         main="Figure 10.1, p438")
axis(1,pos=0,at=2*(1:7))
arrows(0,0,15,0,length=0.1,angle=10)
arrows(0,0,0,0.65,length=0.1,angle=10)
axis(2,pos=0,las=1,at=seq(from=-3,to=6)/10,
         labels=c("-.3","-.2","-.1","0",".1",".2",".3",".4",".5",".6"))
text(0, 0.7, expression(J[n](x)))
text(15.5, 0, expression(x))
text(2.2,0.58,expression(n==0))
text(3.2,0.4,expression(n==1))
text(4.3,0.3,expression(n==2))
text(6.0,0.22,expression(n==3))
```

```
x <- seq(from=0 ,to=5,by=0.1)
cbind(x, bessel_J0(x),bessel_J1(x),bessel_Jn(2,x))      #table 9.1, p390
cbind(x, bessel_Y0(x),bessel_Y1(x),bessel_Yn(2,x))      #table 9.2, p391
t(bessel_Jn_array(3,9,x*2))                             #table 9.2, p398



 x <- seq(from=8,to=10,by=0.2)
 jj <- t(bessel_Jn(n=3:9,x=t(matrix(x,11,7))))
colnames(jj) <- paste("J",3:9,"(x)",sep="")
cbind(x,jj)                #another part of table 9.2, p398


 x <- seq(from=8,to=10,by=0.2)
 jj <- t(bessel_Yn(n=3:9,x=t(matrix(x,11,7))))
colnames(jj) <- paste("J",3:9,"(x)",sep="")
cbind(x,jj)                #part of table 9.2, p399

cbind(                      x,                          #table 9.8, p416
        exp(-x)*bessel_I0  (x),
        exp(-x)*bessel_I1  (x),
         x^(-2)*bessel_In(2,x)
)

cbind(                      x,                          #table 9.8, p417
        exp(x)*bessel_K0  (x),
        exp(x)*bessel_K1  (x),
         x^(2)*bessel_Kn(2,x)
)

cbind(x,                                                #table 10.1 , p457
    bessel_j0(x),
    bessel_j1(x),
    bessel_j2(x),
    bessel_y0(x),
    bessel_y1(x),
    bessel_y2(x)
)

 cbind(0:9,"x=1"=bessel_yl(l=0:9,x=1), "x=2"=bessel_yl(l=0:9,x=2), "x=5"=bessel_yl(l=0:9,x=5))
                                                        #table 10.5, p466, top
```

---

Clausen *Clausen functions*

---

## Description

Clausen functions as per the Gnu Scientific Library section 7.6. These functions are declared in header file `gsl_sf_clausen.h`

### Usage

```
clausen(x, give=FALSE, strict=TRUE)
```

### Arguments

| | |
|---|---|
| x | input: real values |
| give | Boolean with TRUE meaning to return a list of three items: the value, an estimate of the error, and a status number |
| strict | Boolean, with TRUE meaning to return NaN if status is an error |

### Author(s)

Robin K. S. Hankin

### References

https://www.gnu.org/software/gsl/

### Examples

```
x <- (0:30)*pi/180
clausen(x)          #table 27.8, p1006
```

---

## Coulomb *Coulomb functions*

---

### Description

Coulomb functions as per the Gnu Scientific Library, reference manual section 7.7 and AMS-55, chapter 14. These functions are declared in header file gsl_sf_coulomb.h

### Usage

```
hydrogenicR_1(Z, r, give=FALSE, strict=TRUE)
hydrogenicR(n, l, Z, r, give=FALSE, strict=TRUE)
coulomb_wave_FG(eta, x, L_F, k, give=FALSE, strict=TRUE)
coulomb_wave_F_array(L_min, kmax, eta, x, give=FALSE,strict=TRUE)
coulomb_wave_FG_array(L_min, kmax, eta, x, give=FALSE,strict=TRUE)
coulomb_wave_FGp_array(L_min, kmax, eta, x, give=FALSE,strict=TRUE)
coulomb_wave_sphF_array(L_min, kmax, eta, x, give=FALSE,strict=TRUE)
coulomb_CL(L,eta, give=FALSE,strict=TRUE)
coulomb_CL_array(L_min, kmax, eta, give=FALSE,strict=TRUE)
```

## Arguments

| | |
|---|---|
| n,l,kmax | input: integers |
| Z,r,eta,x,L_F,L_min,k,L | input: real values |
| give | Boolean with TRUE meaning to return a list of three items: the value, an estimate of the error, and a status number |
| strict | Boolean, with TRUE meaning to return NaN if status is an error |

## Author(s)

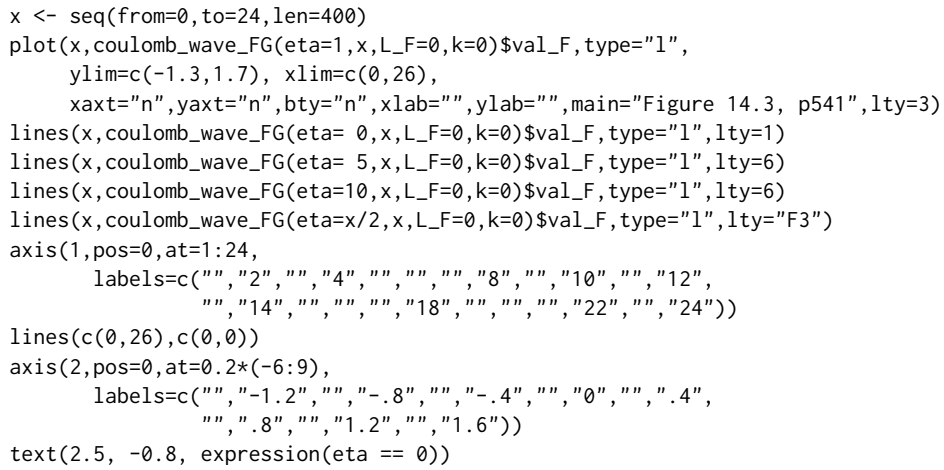
Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
x <- seq(from=0,to=14,len=300)
jj <- coulomb_wave_FG(1,10,x,0)
plot(x,jj$val_F,type="l",xaxt="n",yaxt="n",bty="n",xlab="",ylab="",
       main="Figure 14.1, p539")
lines(x,jj$val_G,type="l",lty=2)
axis(1,pos=0,at=1:14,
       labels=c("","2","","4","","6","","8","","10","","12","","14"))
lines(c(0,1),c(0,0))
axis(2,pos=0)
text(9.5, 0.63, expression(F[L]))
text(8.5, 1.21, expression(G[L]))
```



```
x <- seq(from=0,to=24,len=400)
plot(x,coulomb_wave_FG(eta=1,x,L_F=0,k=0)$val_F,type="l",
     ylim=c(-1.3,1.7), xlim=c(0,26),
     xaxt="n",yaxt="n",bty="n",xlab="",ylab="",main="Figure 14.3, p541",lty=3)
lines(x,coulomb_wave_FG(eta= 0,x,L_F=0,k=0)$val_F,type="l",lty=1)
lines(x,coulomb_wave_FG(eta= 5,x,L_F=0,k=0)$val_F,type="l",lty=6)
lines(x,coulomb_wave_FG(eta=10,x,L_F=0,k=0)$val_F,type="l",lty=6)
lines(x,coulomb_wave_FG(eta=x/2,x,L_F=0,k=0)$val_F,type="l",lty="F3")
axis(1,pos=0,at=1:24,
       labels=c("","2","","4","","","","8","","10","","12",
                "","14","","","","18","","","","22","","24"))
lines(c(0,26),c(0,0))
axis(2,pos=0,at=0.2*(-6:9),
       labels=c("","-1.2","","-.8","","-.4","","0","",".4",
                "",".8","","1.2","","1.6"))
text(2.5, -0.8, expression(eta == 0))
```

```
text(4.5,1.1,adj=0, expression(eta == 1))
text(14,1.4,adj=0, expression(eta == 5))
text(22,1.4,adj=0, expression(eta == 10))
```

```
x <- seq(from=0.5,to=10,by=0.5)
jj <- coulomb_wave_FG(eta=t(matrix(x,20,5)), x=1:5,0,0)
jj.F <- t(jj$val_F)
jj.G <- t(jj$val_G)
colnames(jj.F) <- 1:5
colnames(jj.G) <- 1:5
cbind(x,jj.F)        #table 14.1, p 546, top bit.
cbind(x,jj.G)        #table 14.1, p 547, top bit.
```

---

Coupling *Coupling functions*

---

## Description

Coupling functions as per the Gnu Scientific Library, reference manual section 7.8. These functions are declared in header file `gsl_sf_coupling.h`

## Usage

```
coupling_3j(two_ja, two_jb, two_jc, two_ma, two_mb, two_mc, give=FALSE, strict=TRUE)
coupling_6j(two_ja, two_jb, two_jc, two_jd, two_je, two_jf, give=FALSE, strict=TRUE)
coupling_9j(two_ja, two_jb, two_jc, two_jd, two_je, two_jf,
      two_jg, two_jh, two_ji, give=FALSE, strict=TRUE)
```

## Arguments

| | |
|---|---|
| `two_ja, two_jb, two_jc, two_jd, two_je, two_jf, two_jg, two_jh, two_ji, two_ma, two_mb, two_mc` | Arguments as per the GSL manual |
| `give` | Boolean with `TRUE` meaning to return a list of three items: the value, an estimate of the error, and a status number |
| `strict` | Boolean, with `TRUE` meaning to return `NaN` if status is an error |

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
coupling_3j(1,2,3,4,5,6)
coupling_6j(1,2,3,4,5,6)
coupling_9j(1,2,3,4,5,6,7,8,9)
```

---

Dawson *Dawson functions*

---

## Description

Dawson functions as per the Gnu Scientific Library, reference manual section 7.9. These functions are declared in header file `gsl_sf_dawson.h`

## Usage

```
dawson(x, give=FALSE, strict=TRUE)
```

## Arguments

| | |
|---|---|
| `x` | input: real values |
| `give` | Boolean with `TRUE` meaning to return a list of three items: the value, an estimate of the error, and a status number |
| `strict` | Boolean, with `TRUE` meaning to return `NaN` if status is an error |

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
x <- seq(from=0,to=2,by=0.01)
dawson(x)   #table 7.5 of Ab and St
```

## Debye *Debye functions*

### Description

Debye functions as per the Gnu Scientific Library, section 7.10 of the reference manual. These functions are declared in header file `gsl_sf_debye.h`

### Usage

```
debye_1(x, give=FALSE, strict=TRUE)
debye_2(x, give=FALSE, strict=TRUE)
debye_3(x, give=FALSE, strict=TRUE)
debye_4(x, give=FALSE, strict=TRUE)
```

### Arguments

| | |
|---|---|
| `x` | input: real values |
| `give` | Boolean with `TRUE` meaning to return a list of three items: the value, an estimate of the error, and a status number |
| `strict` | Boolean, with `TRUE` meaning to return `NaN` if status is an error |

### Author(s)

Robin K. S. Hankin

### References

https://www.gnu.org/software/gsl/

### Examples

```
x <- seq(from=0,to=10,by=0.1)
cbind(x,debye_1(x),debye_2(x),debye_3(x),debye_4(x))  #table 27.1
```

## Dilog *Dilog functions*

### Description

Dilog functions as per the Gnu Scientific Library reference manual section 7.11. These functions are declared in header file `gsl_sf_dilog.h`

## Usage

```
dilog(x, give=FALSE, strict=TRUE)
complex_dilog(r, theta, give=FALSE, strict=TRUE)
```

## Arguments

| | |
|---|---|
| x | input: real values |
| r,theta | In complex_dilog(), input values. If theta takes its default value of NULL, interpret r as a complex-valued object. If theta is non-null, interpret r as the Modulus, and theta as the argument, of the complex object passed to gsl_sf_complex_dilog_e() |
| give | Boolean, with default FALSE meaning to return just the answers, and TRUE meaning to return a status vector as well |
| strict | Boolean, with TRUE meaning to return NaN if nonzero status is returned by the GSL function (FALSE means to return the value: use with caution) |

## Details

All functions as documented in the GSL reference manual section 7.11.

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
x <- seq(from=0, to=0.1,by=0.01)
cbind(x,"f(x)"=dilog(1-x))   #table 27.7, p1005
```

---

# Ellint *Elliptic functions*

---

## Description

Elliptic functions as per the Gnu Scientific Library, reference manual section 7.13 and AMS-55, chapter 17. These functions are declared in header file gsl_sf_ellint.h

## Usage

```
ellint_Kcomp(k, mode=0, give=FALSE,strict=TRUE)
ellint_Ecomp(k, mode=0, give=FALSE,strict=TRUE)
ellint_F(phi,k, mode=0, give=FALSE,strict=TRUE)
ellint_E(phi,k, mode=0, give=FALSE,strict=TRUE)
ellint_P(phi,k,n, mode=0, give=FALSE,strict=TRUE)
ellint_D(phi,k, mode=0, give=FALSE,strict=TRUE)
ellint_RC(x, y, mode=0, give=FALSE,strict=TRUE)
ellint_RD(x, y, z, mode=0, give=FALSE,strict=TRUE)
ellint_RF(x, y, z, mode=0, give=FALSE,strict=TRUE)
ellint_RJ(x, y, z, p, mode=0, give=FALSE,strict=TRUE)
```

## Arguments

| | |
|---|---|
| phi,k,n,p,x,y,z | input: real values |
| give | Boolean, with default FALSE meaning to return just the answers, and TRUE meaning to return a status vector as well |
| strict | Boolean |
| mode | input: mode. For GSL_PREC_DOUBLE, GSL_PREC_SINGLE, GSL_PREC_APPROX use 0,1,2 respectively. |

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
ellint_Kcomp(0.3)
ellint_Ecomp(0.3)
ellint_F(0.4,0.7)
ellint_E(0.4,0.7)
ellint_P(0.4,0.7,0.3)
ellint_D(0.4,0.3)
ellint_RC(0.5,0.6)
ellint_RD(0.5,0.6,0.7)
ellint_RF(0.5,0.6,0.7)
ellint_RJ(0.5,0.6,0.7,0.1)


x <- seq(from=0,to=0.5,by=0.01)
col1 <- ellint_Kcomp(sqrt(x))
col2 <- ellint_Kcomp(sqrt(1-x))
col3 <- exp(-pi*col2/col1)
cbind(x,col1,col2,col3)         #table 17.1, p608
```

```
x <- 0:45
col1 <- ellint_Kcomp(sin(pi/180*x))
col2 <- ellint_Kcomp(sin(pi/2-pi/180*x))
col3 <- exp(-pi*col2/col1)
cbind(x,col1,col2,col3)       #table 17.2, p610

x <- seq(from=0,to=90,by=2)
f <- function(a){ellint_F(phi=a*pi/180,sin(x*pi/180))}
g <- function(a){ellint_E(phi=a*pi/180,sin(x*pi/180))}
h <- function(a,n){ellint_P(phi=a*pi/180,sin( a*15*pi/180),n)}
i <- function(x){ellint_P(phi=x*pi/180, k=sin((0:6)*15*pi/180),  n= -0.6)}


cbind(x,f(5),f(10),f(15),f(20),f(25),f(30))          #table 17.5, p613
cbind(x,g(5),g(10),g(15),g(20),g(25),g(30))          #table 17.6, p616



cbind(i(15),i(30),i(45),i(60),i(75),i(90))           #table 17.9,
                                                     #(BOTTOM OF p625)
```

---

`Elljac` *Elliptic functions*

---

## Description

Elljac functions as per the Gnu Scientific Library, reference manual section 7.14 and AMS-55, chapter 16. These functions are declared in header file `gsl_sf_elljac.h`

## Usage

```
elljac(u, m, give=FALSE, strict=TRUE)
gsl_sn(z,m)
gsl_cn(z,m)
gsl_dn(z,m)
gsl_ns(z,m)
gsl_nc(z,m)
gsl_nd(z,m)
gsl_sc(z,m)
gsl_sd(z,m)
gsl_cs(z,m)
gsl_cd(z,m)
gsl_ds(z,m)
gsl_dc(z,m)
```

## Arguments

| | |
|---|---|
| u,m | input: real values |
| z | input: complex values |
| give | Boolean with TRUE meaning to return a list of three items: the value, an estimate of the error, and a status number |
| strict | Boolean, with TRUE meaning to return NaN if status is an error |

## Details

A straightforward wrapper for the gsl_sf_elljac_e function of the GSL library, except for gsl_sn(), gsl_cn(), and gsl_dn(), which implement 16.21.1 to 16.21.4 (thus taking complex arguments); and gsl_ns() et seq which are the minor elliptic functions.

Function sn_cn_dn() is not really intended for the end-user.

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
K <- ellint_F(phi=pi/2,k=sqrt(1/2))  #note the sqrt: m=k^2
u <- seq(from=0,to=4*K,by=K/24)
jj <- elljac(u,1/2)
plot(u,jj$sn,type="l",xaxt="n",yaxt="n",bty="n",ylab="",xlab="",main="Fig 16.1, p570")
lines(u,jj$cn,lty=2)
lines(u,jj$dn,lty=3)
axis(1,pos=0,at=c(K,2*K,3*K,4*K),labels=c("K","2K","3K","4K"))
abline(0,0)
axis(2,pos=0,at=c(-1,1))
text(1.8*K,0.6,"sn u")
text(1.6*K,-0.5,"cn u")
text(2.6*K,0.9,"dn u")



 a <- seq(from=-5,to=5,len=100)
jj <- outer(a,a,function(a,b){a})
z <- jj+1i*t(jj)
e <- Re(gsl_cd(z,m=0.2))
e[abs(e)>10] <- NA
contour(a,a,e,nlev=55)
```

Error *Error functions*

## Description

Error functions as per the Gnu Scientific Library, reference manual section 7.15 and AMS-55, chapter 7. Thes functions are declared in header file `gsl_sf_error.h`

## Usage

```
erf(x, mode=0, give=FALSE, strict=TRUE)
erfc(x, mode=0, give=FALSE, strict=TRUE)
log_erfc(x, mode=0, give=FALSE, strict=TRUE)
erf_Q(x, mode=0, give=FALSE, strict=TRUE)
hazard(x, mode=0, give=FALSE, strict=TRUE)
```

## Arguments

| | |
|---|---|
| `x` | input: real values |
| `give` | Boolean with `TRUE` meaning to return a list of three items: the value, an estimate of the error, and a status number |
| `mode` | input: mode. For `GSL_PREC_DOUBLE` , `GSL_PREC_SINGLE`, `GSL_PREC_APPROX` use `0`,`1`,`2` respectively |
| `strict` | Boolean, with `TRUE` meaning to return `NaN` if status is an error |

## Details

The zero functions return a status of `GSL_EDOM` and a value of `NA` for $n \leq 0$

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
erf(0.745) # Example 1, page 304
```

`Expint` *exponential functions*

## Description

Expint functions as per the Gnu Scientific Library, reference manual section 7.17 and AMS-55, chapter 5. These functions are declared in header file `gsl_sf_expint.h`.

## Usage

```
expint_E1(x, give=FALSE, strict=TRUE)
expint_E2(x, give=FALSE, strict=TRUE)
expint_En(n, x, give=FALSE, strict=TRUE)
expint_Ei(x, give=FALSE, strict=TRUE)
Shi(x, give=FALSE, strict=TRUE)
Chi(x, give=FALSE, strict=TRUE)
expint_3(x, give=FALSE, strict=TRUE)
Si(x, give=FALSE, strict=TRUE)
Ci(x, give=FALSE, strict=TRUE)
atanint(x, give=FALSE, strict=TRUE)
```

## Arguments

| | |
|---|---|
| `x` | input: real values |
| `n` | input: integer values |
| `give` | Boolean with `TRUE` meaning to return a list of three items: the value, an estimate of the error, and a status number |
| `strict` | Boolean, with `TRUE` meaning to return `NaN` if status is an error |

## Note

Function `expint_En()` requires GSL version 1.8 or later.

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
x <- seq(from=0.5, to=1, by=0.01)
cbind(x,Si(x),Ci(x),expint_Ei(x),expint_E1(x))  #table 5.1 of AS, p239

x <- seq(from=0, to=12, len=100)
plot(x,Ci(x),col="black",type="l",xaxt="n",yaxt="n",bty="n",
         xlab="",ylab="",main="Figure 5.6, p232",
         xlim=c(0,12),ylim=c(-1,2.0))
lines(x,Si(x))
axis(1,pos=0)
axis(2,pos=0)
 abline(h=pi/2,lty=2)


# Table 5.4, page 245:
 xvec <- seq(from=0,by=0.01,len=20)
 nvec <- c(2,3,4,10,20)
 x <- kronecker(xvec,t(rep(1,5)))
 n <- kronecker(t(nvec),rep(1,20))
 ans <- cbind(x=xvec,expint_En(n,x))
 rownames(ans) <- rep(" ",length(xvec))
 colnames(ans) <- c("x",paste("n=",nvec,sep=""))
 class(ans) <- "I do not understand the first column"

 ans
```

---

`Fermi-Dirac` *Fermi-Dirac functions*

---

## Description

Fermi-Dirac functions as per the Gnu Scientific Library, reference manual section 7.18. These functions are declared in header file `gsl_sf_fermi_dirac.h`

## Usage

```
fermi_dirac_m1(x, give=FALSE, strict=TRUE)
fermi_dirac_0(x, give=FALSE, strict=TRUE)
fermi_dirac_1(x, give=FALSE, strict=TRUE)
fermi_dirac_2(x, give=FALSE, strict=TRUE)
fermi_dirac_int(j, x, give=FALSE, strict=TRUE)
fermi_dirac_mhalf(x, give=FALSE, strict=TRUE)
fermi_dirac_half(x, give=FALSE, strict=TRUE)
fermi_dirac_3half(x, give=FALSE, strict=TRUE)
fermi_dirac_inc_0(x, b, give=FALSE, strict=TRUE)
```

## Arguments

| | |
|---|---|
| x,j,b | input: real values |
| give | Boolean with TRUE meaning to return a list of three items: the value, an estimate of the error, and a status number |
| strict | Boolean, with TRUE meaning to return NaN if status is an error |

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
x <- seq(from=0,to=2,by=0.01)
fermi_dirac_m1(x)   #table 7.5 of Ab and St
```

---

Gamma *gamma functions*

---

## Description

Gamma functions as per the Gnu Scientific Library reference manual section 7.19. These functions are declared in header file gsl_sf_gamma.h

## Usage

```
gsl_sf_gamma(x,give=FALSE,strict=TRUE)
lngamma(x,give=FALSE,strict=TRUE)
lngamma_sgn(x,give=FALSE,strict=TRUE)
gammastar(x,give=FALSE,strict=TRUE)
gammainv(x,give=FALSE,strict=TRUE)
lngamma_complex(zr, zi=NULL, r.and.i=TRUE, give=FALSE, strict=TRUE)
taylorcoeff(n,x,give=FALSE,strict=TRUE)
fact(n,give=FALSE,strict=TRUE)
doublefact(n,give=FALSE,strict=TRUE)
lnfact(n,give=FALSE,strict=TRUE)
lndoublefact(n,give=FALSE,strict=TRUE)
gsl_sf_choose(n,m,give=FALSE,strict=TRUE)
lnchoose(n,m,give=FALSE,strict=TRUE)
poch(a,x,give=FALSE,strict=TRUE)
lnpoch(a,x,give=FALSE,strict=TRUE)
lnpoch_sgn(a,x,give=FALSE,strict=TRUE)
pochrel(a,x,give=FALSE,strict=TRUE)
```

```
gamma_inc_Q(a,x,give=FALSE,strict=TRUE)
gamma_inc_P(a,x,give=FALSE,strict=TRUE)
gamma_inc(a,x,give=FALSE,strict=TRUE)
gsl_sf_beta(a,b,give=FALSE,strict=TRUE)
lnbeta(a,b,give=FALSE,strict=TRUE)
beta_inc(a,b,x,give=FALSE,strict=TRUE)
```

## Arguments

| | |
|---|---|
| x,a,b | input: real values |
| m,n | input: integer value |
| zr | In gamma_complex(), the real part of the argument |
| zi | In gamma_complex(), the imaginary part of the argument. If missing (ie takes the default value of NULL), interpret zr as complex, even if real |
| r.and.i | In gamma_complex(), Boolean variable with default value of TRUE meaning to return a complex variable as per the details section below; and FALSE meaning to return the values as advertised in the GSL manual |
| give | Boolean with TRUE meaning to return a list of three items: the value, an estimate of the error, and a status number |
| strict | Boolean, with TRUE meaning to return NaN if status is an error |

## Details

All functions as documented in the GSL reference manual section 7.19.

Note that gamma_inc_P() gives the area of the left tail of the gamma distribution so, for example, gamma_inc_P(1.8, 5) = pgamma(5, 1.8) to numerical accuracy.

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
gsl_sf_gamma(3)

lngamma_complex(1+seq(from=0,to=5,by=0.1)*1i)  #table 6.7, p 277 (LH col)
                                               #note 2pi phase diff


jj <- expand.grid(1:10,2:5)
x <- taylorcoeff(jj$Var1,jj$Var2)
dim(x) <- c(10,4)
x    #table 23.5, p818
```

```
jj <- expand.grid(36:50,9:13)
x <- gsl_sf_choose(jj$Var1,jj$Var2)
dim(x) <- c(15,5)
x     #table 24.1, p829  (bottom bit)

gamma_inc(1.2,1.3)
beta(1.2, 1.3)
lnbeta(1.2,1.55)
beta_inc(1.2,1.4,1.6)

gamma_inc_P(1.8, 5) - pgamma(5, 1.8)  # should be small
```

---

Gegenbauer *Gegenbauer functions*

---

## Description

Gegenbauer functions as per the Gnu Scientific Library reference manual section 7.20, and AMS-55, chapter 22. These functions are declared in header file `gsl_sf_gegenbauer.h`

## Usage

```
gegenpoly_1(lambda, x, give=FALSE,strict=TRUE)
gegenpoly_2(lambda, x, give=FALSE,strict=TRUE)
gegenpoly_3(lambda, x, give=FALSE,strict=TRUE)
gegenpoly_n(n,lambda, x, give=FALSE,strict=TRUE)
gegenpoly_array(nmax,lambda, x, give=FALSE,strict=TRUE)
```

## Arguments

| | |
|---|---|
| `lambda,x` | input: real values |
| `n,nmax` | input: integer value |
| `give` | Boolean with `TRUE` meaning to return a list of three items: the value, an estimate of the error, and a status number |
| `strict` | Boolean, with `TRUE` meaning to return `NaN` if status is an error |

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
x <- seq(from=-1 ,to=1,len=300)
y <- gegenpoly_array(6,0.5,x)
matplot(x,t(y[-(1:2),]), xlim=c(-1,1.2),ylim=c(-0.5,1.5),
       type="l",xaxt="n",yaxt="n",bty="n",xlab="",ylab="",
       main="Figure 22.5, p777",col="black")
axis(1,pos=0)
axis(2,pos=0)


plot(x, gegenpoly_n(5,lambda=0.2, x,give=FALSE,strict=TRUE),
xlim=c(-1,1),ylim=c(-1.5,1.5),main="Figure 22.5, p777",
type="n",xaxt="n",yaxt="n",bty="n",xlab="",ylab="")
lines(x, gegenpoly_n(5,lambda=0.2, x,give=FALSE,strict=TRUE))
lines(x, gegenpoly_n(5,lambda=0.4, x,give=FALSE,strict=TRUE))
lines(x, gegenpoly_n(5,lambda=0.6, x,give=FALSE,strict=TRUE))
lines(x, gegenpoly_n(5,lambda=0.8, x,give=FALSE,strict=TRUE))
lines(x, gegenpoly_n(5,lambda=1.0, x,give=FALSE,strict=TRUE))
axis(1,pos=0)
axis(2,pos=0,las=1)
```

---

`gsl-deprecated` *gsl-deprecated*

---

## Description

Deprecated Legendre functions as per the Gnu Scientific Library reference manual section 7.24.

## Usage

```
legendre_Plm_array(...)
legendre_Plm_deriv_array(...)
legendre_sphPlm_array(...)
legendre_sphPlm_deriv_array(...)
legendre_array_size(...)
deprecated_legendre(...)
```

## Arguments

| | |
|---|---|
| `...` | (ignored) |

## Note

As of GSL-2.1, functions

- gsl_sf_legendre_Plm_array
- gsl_sf_legendre_Plm_deriv_array
- gsl_sf_legendre_sphPlm_array
- gsl_sf_legendre_sphPlm_deriv_array
- gsl_sf_legendre_array_size

are deprecated. This functionality is now provided in GSL by the gsl_sf_legendre_array suite of functions; in R, use one of:

- legendre_array()
- legendre_deriv_array()
- legendre_deriv_alt_array()
- legendre_deriv2_array()
- legendre_deriv2_alt_array().

These are documented under ?Legendre.

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## See Also

Legendre

---

# Hyperg *Hypergeometric functions*

---

## Description

Hypergeometric functions as per the Gnu Scientific Library reference manual section 7.21 and AMS-55, chapters 13 and 15. These functions are declared in header file gsl_sf_hyperg.h

## Usage

```
hyperg_0F1(c, x, give=FALSE, strict=TRUE)
hyperg_1F1_int(m, n, x, give=FALSE, strict=TRUE)
hyperg_1F1(a, b, x, give=FALSE, strict=TRUE)
hyperg_U_int(m, n, x, give=FALSE, strict=TRUE)
hyperg_U(a, b, x, give=FALSE, strict=TRUE)
hyperg_2F1(a, b, c, x, give=FALSE, strict=TRUE)
hyperg_2F1_conj(aR, aI, c, x, give=FALSE, strict=TRUE)
hyperg_2F1_renorm(a, b, c, x, give=FALSE, strict=TRUE)
hyperg_2F1_conj_renorm(aR, aI, c, x, give=FALSE, strict=TRUE)
hyperg_2F0(a, b, x, give=FALSE, strict=TRUE)
```

## Arguments

| | |
|---|---|
| x | input: real values |
| a,b,c | input: real values |
| m,n | input: integer values |
| aR,aI | input: real values |
| give | Boolean with TRUE meaning to return a list of three items: the value, an estimate of the error, and a status number. |
| strict | Boolean, with TRUE meaning to return NaN if status is an error |

## Note

"The circle of convergence of the Gauss hypergeometric series is the unit circle $|z| = 1$" (AMS, page 556).

There is a known issue in hyperg_2F1() in GSL-2.6, https://savannah.gnu.org/bugs/?54998 and the package returns the erroneous value given by GSL.

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
hyperg_0F1(0.1,0.55)

hyperg_1F1_int(2,3,0.555)
hyperg_1F1(2.12312,3.12313,0.555)
hyperg_U_int(2, 3, 0.555)
hyperg_U(2.234, 3.234, 0.555)
```

Laguerre *Laguerre functions*

## Description

Laguerre functions as per the Gnu Scientific Library reference manual section 7.22. These functions are declared in header file `gsl_sf_laguerre.h`

## Usage

```
laguerre_1(a, x, give=FALSE, strict=TRUE)
laguerre_2(a, x, give=FALSE, strict=TRUE)
laguerre_3(a, x, give=FALSE, strict=TRUE)
laguerre_n(n, a, x, give=FALSE, strict=TRUE)
```

## Arguments

| | |
|---|---|
| `a,x` | input: real values |
| `n` | input: integer values |
| `give` | Boolean with `TRUE` meaning to return a list of three items: the value, an estimate of the error, and a status number |
| `strict` | Boolean, with `TRUE` meaning to return `NaN` if status is an error |

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
x <- seq(from=0,to=6,len=100)
plot(x,laguerre_n(2,0,x),xlim=c(0,6),ylim=c(-2,3),
          type="l",xaxt="n",yaxt="n",bty="n",xlab="",ylab="",
          main="Figure 22.9, p780")

lines(x,laguerre_n(3,0,x))
lines(x,laguerre_n(4,0,x))
lines(x,laguerre_n(5,0,x))
axis(1,pos=0)
axis(2,pos=0)
```

## Lambert *Lambert's W function*

### Description

Lambert's W function as per the Gnu Scientific Library reference manual section 7.23. These functions are declared in header file `gsl_sf_lambert.h`

### Usage

```
lambert_W0(x, give=FALSE, strict=TRUE)
lambert_Wm1(x, give=FALSE,strict=TRUE)
```

### Arguments

| | |
|---|---|
| `x` | input: real values |
| `give` | Boolean with `TRUE` meaning to return a list of three items: the value, an estimate of the error, and a status number |
| `strict` | Boolean, with `TRUE` meaning to return `NaN` if status is an error |

### Author(s)

Robin K. S. Hankin

### References

https://www.gnu.org/software/gsl/

### Examples

```
a <- runif(6)
L <- lambert_W0(a)
print(L*exp(L) - a)
```

## Legendre *Legendre functions*

### Description

Legendre functions as per the Gnu Scientific Library reference manual section 7.24, and AMS-55, chapter 8. These functions are declared in header file `gsl_sf_legendre.h`

## Usage

```
legendre_P1(x, give=FALSE, strict=TRUE)
legendre_P2(x, give=FALSE, strict=TRUE)
legendre_P3(x, give=FALSE, strict=TRUE)
legendre_Pl(l, x, give=FALSE, strict=TRUE)
legendre_Pl_array(lmax, x, give=FALSE, strict=TRUE)
legendre_Q0(x, give=FALSE, strict=TRUE)
legendre_Q1(x, give=FALSE, strict=TRUE)
legendre_Ql(l, x, give=FALSE, strict=TRUE)
legendre_array_n(lmax)
legendre_array_index(l,m)
legendre_check_args(x,lmax,norm,csphase)
legendre_array(x, lmax, norm=1, csphase= -1)
legendre_deriv_array(x, lmax, norm=1, csphase= -1)
legendre_deriv_alt_array(x, lmax, norm=1, csphase= -1)
legendre_deriv2_array(x, lmax, norm=1, csphase= -1)
legendre_deriv2_alt_array(x, lmax, norm=1, csphase= -1)
legendre_Plm(l, m, x, give=FALSE, strict=TRUE)
legendre_sphPlm(l, m, x, give=FALSE, strict=TRUE)
conicalP_half(lambda, x, give=FALSE, strict=TRUE)
conicalP_mhalf(lambda, x, give=FALSE, strict=TRUE)
conicalP_0(lambda, x, give=FALSE, strict=TRUE)
conicalP_1(lambda, x, give=FALSE, strict=TRUE)
conicalP_sph_reg(l, lambda, x, give=FALSE, strict=TRUE)
conicalP_cyl_reg(m, lambda, x, give=FALSE, strict=TRUE)
legendre_H3d_0(lambda, eta, give=FALSE, strict=TRUE)
legendre_H3d_1(lambda, eta, give=FALSE, strict=TRUE)
legendre_H3d(l, lambda, eta, give=FALSE, strict=TRUE)
legendre_H3d_array(lmax, lambda, eta, give=FALSE, strict=TRUE)
```

## Arguments

| | |
|---|---|
| `eta,lambda,x` | input: real values |
| `l,m,lmax` | input: integer values |
| `csphase,norm` | Options for use with `legendre_array()` |
| `give` | Boolean, with default `FALSE` meaning to return just the answers, and `TRUE` meaning to return a status vector as well |
| `strict` | Boolean, with `TRUE` meaning to return `NaN` if nonzero status is returned by the GSL function (`FALSE` means to return the value: use with caution) |

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
 theta <- seq(from=0,to=pi/2,len=100)
 plot(theta,legendre_P1(cos(theta)),type="l",ylim=c(-0.5,1), main="Figure 8.1, p338")
 abline(1,0)
 lines(theta,legendre_P2(cos(theta)),type="l")
 lines(theta,legendre_P3(cos(theta)),type="l")

x <- seq(from=0,to=1,len=600)
plot(x, legendre_Plm(3,1,x), type="l",lty=3,main="Figure 8.2, p338: note sign error")
lines(x,legendre_Plm(2,1,x), type="l",lty=2)
lines(x,legendre_Plm(1,1,x), type="l",lty=1)
abline(0,0)


plot(x,legendre_Ql(0,x),xlim=c(0,1), ylim=c(-1,1.5), type="l",lty=1,
main="Figure 8.4, p339")
lines(x,legendre_Ql(1,x),lty=2)
lines(x,legendre_Ql(2,x),lty=3)
lines(x,legendre_Ql(3,x),lty=4)
abline(0,0)

#table 8.1 of A&S:
t(legendre_Pl_array(10, seq(from=0,to=1,by=0.01))[1+c(2,3,9,10),])

#table 8.3:
f <- function(n){legendre_Ql(n, seq(from=0,to=1,by=0.01))}
sapply(c(0,1,2,3,9,10),f)


# Some checks for the legendre_array() series:

# P_6^1(0.3):
legendre_array(0.3,7)[7,2]         # MMA:  LegendreP[6,1,0.3]; note off-by-one issue

# d/dx  P_8^5(x) @ x=0.2:
legendre_deriv_array(0.2,8)[9,6]   # MMA: D[LegendreP[8,5,x],x] /. {x -> 0.2}


# alternative derivatives:
 legendre_deriv_alt_array(0.4,8)[9,6]  # D[LegendreP[8,5,Cos[x]],x] /. x -> ArcCos[0.4]
```

---

Log *Log functions*

---

## Description

Log functions as per the Gnu Scientific Library, reference manual section 7.25 and AMS-55, chapter 4. These functions are declared in header file `gsl_sf_log.h`

## Usage

```
gsl_sf_log(x, give=FALSE, strict=TRUE)
log_abs(x, give=FALSE, strict=TRUE)
complex_log(zr, zi=NULL, r.and.i=TRUE, give=FALSE, strict=TRUE)
log_1plusx(x, give=FALSE, strict=TRUE)
log_1plusx_mx(x, give=FALSE, strict=TRUE)
```

## Arguments

| | |
|---|---|
| x | input: real values |
| zr | In complex_log(), the real part of the argument |
| zi | In complex_log(), the imaginary part of the argument. If missing (ie takes the default value of NULL), interpret zr as complex, even if real |
| r.and.i | In complex_log(), Boolean variable with default value of TRUE meaning to return a complex variable as per the details section below; and FALSE meaning to return the values as advertised in the GSL manual |
| give | Boolean with TRUE meaning to return a list of three items: the value, an estimate of the error, and a status number |
| strict | Boolean, with TRUE meaning to return NaN if status is an error |

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
x <- seq(from=0.1,to=2,by=0.01)
log(x)   #table 7.5 of Ab and St
```

---

# Misc *Argument processing and general info*

---

## Description

Various widely used functions in the package

## Usage

```
process.args(...)
strictify(val,status)
```

## Arguments

| | |
|---|---|
| ... | Argument list to be coerced to the same length |
| val | Value component of &result |
| status | status integer |

## Details

Function process.args() is an internal function used to massage the arguments into a form suitable for passing to .C(). For example, in function hyperg_0F1(c,x), one wants each of hyperg_0F1(0.1, c(0.3,0.4)) and hyperg_0F1(c(0.1,0.2), 0.3) and hyperg_0F1(c(0.1,0.2),c(0.3,0.4)) to behave sensibly.

Function process.args() is used widely in the package, taking an arbitrary number of arguments and returning a list whose elements are vectors of the same length. Most of the special functions use process.args() to ensure that the returned value takes the attributes of the input argument with most elements where possible.

Function strictify() uses the status value returned by the "error" form of the GSL special functions to make values returned with a nonzero error a NaN. In most of the special functions, strictify() is called if argument strict takes its default value of TRUE. Setting it to FALSE sometimes returns a numerical value as per the GSL reference manual.

In most of the special functions, if argument give takes its default value of FALSE, only a numerical value is returned. If TRUE, error information and the status (see preceding paragraph) is also returned.

Following tips found on R-devel:

1. Download and extract source code of R-package **gsl**
2. Use gsl-config --libs to get the path to GSL's lib directory (-L<path-to-lib>), use gsl-config --cflags to get the path to GSL's include directory (-I<path-to-include>)
3. Change Makevars in gsl/src:
   - Add -L<path-to-lib> to PKG_LIBS
   - Add (new) line: PKG_CPPFLAGS=-I<path-to-include>
4. Install gsl via

   LDFLAGS=-L<path-to-lib>; export LDFLAGS

   CPPFLAGS=-I<path-to-include>;export CPPFLAGS

   R CMD INSTALL gsl

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

`multimin` *Function minimization*

## Description

*These functions have been removed from the package temporarily, pending a permanent fix.*

Function minimization using the Gnu Scientific Library, reference manual section 35. These functions are declared in header file `gsl_multimin.h`

Several algorithms for finding (local) minima of functions in one or more variables are provided. All of the algorithms operate locally, in the sense that they maintain a best guess and require the function to be continuous. Apart from the Nelder-Mead algorithm, these algorithms also use a derivative.

## Usage

```
multimin(..., prec=0.0001)
multimin.init(x, f, df=NA, fdf=NA, method=NA, step.size=NA, tol=NA)
multimin.iterate(state)
multimin.restart(state)
multimin.fminimizer.size(state)
```

## Arguments

| | |
|---|---|
| `...` | In function `multimin()`, the argument list passed to `multimin.init()` |
| `x` | A starting point. These algorithms are faster with better initial guesses |
| `f` | The function to minimize. This function must take a single `numeric` vector as input, and output a `numeric` scalar |
| `df` | The derivative of `f`. This is required for all algorithms except Nelder-Mead |
| `fdf` | A function that evaluates `f` and `df` simultaneously. This is optional, and is only useful if simultaneous evaluation is faster |
| `method` | The algorithm to use, which is one of "`conjugate-fr`", "`conjugate-pr`", "`bfgs`", "`steepest-descent`" and "`nm`" |
| `step.size` | This step size guides the algorithm to pick a good distance between points in its search |
| `tol` | This parameter is relevant for gradient-based methods. It controls how much the gradient should flatten out in each line search. More specifically, let $u(t) = f(x + st)$ be the function restricted to the search ray. Then a point $t$ is tolerable if $u'(t) < tolu'(0)$. Higher values give more lax linesearches. This parameter trades-off searching intensively in the outer loop (finding search directions) versus the inner loop (finding a good point in a particular direction) |
| `prec` | The stopping-rule precision parameter. For the derivative-based methods, a solution is good enough if the norm of the gradient is smaller than `prec`. For the non-derivative-based methods, a solution is good enough if the norm of successive solutions is smaller than `prec` |
| `state` | This stores all information relating to the progress of the optimization problem |

## Details

There are two ways to call `multimin`. The simple way is to merely call `multimin` directly. A more complicated way is to call `multimin.init` first, and then repeatedly call `multimin.iterate` until the guess gets good enough. In addition, `multimin.restart` can be used with the second approach to discard accumulated information (such as curvature information) if that information turns out to be unhelpful. This is roughly equivalent to calling `multimin.init` by setting the starting point to be the current best guess.

All of the derivative-based methods consist of iterations that pick a descent direction, and conduct a line search for a better point along the ray in that direction from the current point. The Fletcher-Reeves and Polak-Ribiere conjugate gradient algorithms maintain a a vector that summarizes the curvature at that point. These are useful for high-dimensional problems (eg: more than 100 dimensions) because they don't use matrices which become expensive to keep track of. The Broyden-Fletcher-Goldfarb-Shanno is better for low-dimensional problems, since it maintains an approximation of the Hessian of the function as well, which gives better curvature information. The steepest-descent algorithm is a naive algorithm that does not use any curvature information. The Nelder-Mead algorithm which does not use derivatives.

## Value

All of these functions return a state variable, which consists of the following items:

| | |
|---|---|
| `internal.state` | Bureaucratic stuff for communicating with GSL |
| `x` | The current best guess of the optimal solution |
| `f` | The value of the function at the best guess |
| `df` | The derivative of the function at the best guess |
| `is.fdf` | TRUE if the algorithm is using a derivative |
| `code` | The GSL return code from the last iteration |

## Note

The source code for the functions documented here conditionalizes on `WIN32`; under windows there is a slight memory leak.

## Author(s)

Andrew Clausen <clausen@econ.upenn.edu>

## References

https://www.gnu.org/software/gsl/

## See Also

`optim` and `nlm` are the standard optimization functions in R.

`deriv` and `D` are the standard symbolic differentation functions in R. `Ryacas` provides more extensive differentiation support using Yet Another Computer Algebra System.

numericDeriv is the standard numerical differentation function in R. GSL can also do numerical differentiation, but no-one has written an R interface yet.

multimin requires the objective function to have a single (vector) argument. unlist and relist are useful for converting between more convenient forms.

## Examples

```
# COMMENTED OUT PENDING PERMANENT FIX
# The Rosenbrock function:

# x0 <- c(-1.2, 1)
# f <- function(x) (1 - x[1])^2 + 100 * (x[2] - x[1]^2)^2
# df <- function(x) c(-2*(1 - x[1]) + 100 * 2 * (x[2] - x[1]^2) * (-2*x[1]),
#                     100 * 2 * (x[2] - x[1]^2))
#
# # The simple way to call multimin.
# state <- multimin(x0, f, df)
# print(state$x)
#
# # The fine-control way to call multimin.
# state <- multimin.init(x0, f, df, method="conjugate-fr")
# for (i in 1:200)
#  state <- multimin.iterate(state)
# print(state$x)
```

---

# Poly *Polynomials*

---

## Description

Polynomial functions as per the Gnu Scientific Library, reference manual section 6.1. These functions are defined in header file gsl_poly.h

## Usage

```
gsl_poly(c_gsl,x)
```

## Arguments

| | |
|---|---|
| c_gsl | Coefficients of the poynomial (c in the function definition and the GSL ref manual) starting at the constant term and ending in the highest power; see details section. This argument is called "c_gsl" (and not "c") to avoid confusion with R function c() |
| x | input: real values |

## Details

One must be careful to avoid off-by-one errors. In C idiom, the function evaluates the polynomial

$$c[0] + c[1]x + c[2]x^2 + \ldots + c[\text{len} - 1]x^{\text{len}-1}$$

where len is the second argument of GSL function `gsl_poly_eval()`.

The R idiom would be

$$c[1] + c[2]x + c[3]x^2 + \ldots + c[\text{len}]x^{\text{len}-1}.$$

This section is work-in-progress and more will be added when I have the time/need for the other functions here.

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
a <- matrix(1:4,2,2)
rownames(a) <- letters[1:2]
(jj <- gsl_poly(1:3,a))

jj-(1 + 2*a + 3*a^2)  #should be small
```

---

`Powint` *Power functions*

---

## Description

Power functions as per the Gnu Scientific Library reference manual section 7.27. These functions are declared in the header file `gsl_sf_pow_int.h`

## Usage

```
pow_int(x, n, give=FALSE, strict=TRUE)
```

## Arguments

| | |
|---|---|
| `x` | input: real values |
| `n` | input: integer values |
| `give` | Boolean with `TRUE` meaning to return a list of three items: the value, an estimate of the error, and a status number |
| `strict` | Boolean, with `TRUE` meaning to return `NaN` if status is an error |

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
pow_int(pi/2,1:10)
```

---

Psi *Psi (digamma) functions*

---

## Description

Psi (digamma) functions as per the Gnu Scientific Library, reference manual section 7.27. These functions are declared in header file `gsl_sf_psi.h`

## Usage

```
psi_int(n, give=FALSE, strict=TRUE)
psi(x, give=FALSE, strict=TRUE)
psi_1piy(y, give=FALSE, strict=TRUE)
psi_1_int(n, give=FALSE, strict=TRUE)
psi_1(x, give=FALSE, strict=TRUE)
psi_n(m, x, give=FALSE, strict=TRUE)
```

## Arguments

| | |
|---|---|
| `m,n` | input: integer values |
| `x,y` | input: real values |
| `give` | Boolean with `TRUE` meaning to return a list of three items: the value, an estimate of the error, and a status number |
| `strict` | Boolean, with default `TRUE` meaning to return `NaN` if status is an error |

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
x <- seq(from=1.2,to=1.25,by=0.005)
cbind(x,psi(x),psi_1(x))
#tabe 6.1, p267, bottom bit

psi_int(1:6)
psi(pi+(1:6))
psi_1piy(pi+(1:6))
psi_1_int(1:6)
psi_n(m=5,x=c(1.123,1.6523))
```

---

Qrng *Quasi-random sequences*

---

## Description

Quasi-random sequences as per the Gnu Scientific Library, reference manual section 18. These functions are declared in header file `gsl_qrng.h`

## Usage

```
qrng_alloc(type = c("niederreiter_2", "sobol"), dim)
qrng_clone(q)
qrng_init(q)
qrng_name(q)
qrng_size(q)
qrng_get(q, n = 1)
```

## Arguments

| | |
|---|---|
| `type` | Type of sequence |
| `dim` | Dimension of sequence |
| `q` | Generator from `qrng_alloc` or `qrng_clone` |
| `n` | How many vectors to generate |

## Details

These are wrappers for the quasi-random sequence functions from the GSL https://www.gnu.org/software/gsl/ with arguments corresponding to those from the library, with a few exceptions. In particular: I have used `dim` where the GSL uses just `d`; I have added the `n` argument to the `qrng_get` function, so that a single call can generate `n` vectors; I have not provided R functions corresponding to `qrng_free` (because R will automatically free the generator when it is garbage collected) or `qrng_state` or `qrng_memcpy` (because these don't make sense within R.)

## Value

qrng_alloc, qrng_clone and qrng_init return an external pointer to the C structure representing the generator. The internals of this structure are not accessible from within R.

qrng_name returns a character vector giving the name of the generator.

qrng_size returns an integer value giving the internal memory usage of the generator.

qrng_get returns a matrix with n rows and dim columns. Each row is a vector in the quasi-random sequence.

## Author(s)

Duncan Murdoch

## References

https://www.gnu.org/software/gsl/

## Examples

```
q <- qrng_alloc(dim = 2)
qrng_name(q)
qrng_get(q, 10)
```

---

# Rng *Random numbers generation*

---

## Description

Random number generation with the Gnu Scientific Library, as per the reference manual section 17

## Usage

```
rng_alloc(type)
rng_clone(r)
rng_name(r)
rng_max(r)
rng_min(r)
rng_set(r, seed)
rng_get(r, length)
rng_uniform(r, length)
rng_uniform_int(r, N, length)
rng_uniform_pos(r, length)
```

### Arguments

| | |
|---|---|
| type | In function rng_alloc(), type of random number generator. This argument is taken to be a character string which is matched to the names of the random number generators given in the GSL manual section 17.9, with the initial "gsl_rng_" removed (for example, to use generator gsl_rng_ranlux, set type to ranlux). Partial matching is used; a null string is interpreted as mt19937. |
| r | Instance of a random number generator. Generate this using function rng_alloc(). |
| seed | Random number seed |
| length | Length of vector of random numbers to create |
| N | In function rng_uniform_int(), upper bound of uniform distribution |

### Details

These are wrappers for the random number generator functions from the GSL https://www.gnu.org/software/gsl/ with arguments corresponding to those from the library. Calling rng_free is not necessary as R performs garbage collection automatically.

The functions that return random numbers (rng_get, rng_uniform, rng_uniform_int, rng_uniform_pos) take an extra argument that specifies the length of the vector of random numbers to be returned.

### Value

Function rng_alloc() returns an external pointer to a GSL random number generator.

### Author(s)

Max Bruche

### References

https://www.gnu.org/software/gsl/

### Examples

```
r <- rng_alloc("cmrg")
rng_set(r, 100)
rng_uniform(r, 10)
```

---

## Synchrotron *Synchrotron functions*

---

### Description

Synchrotron functions as per the Gnu Scientific Library, reference section 7.29. These functions are declared in header file gsl_sf_synchrotron.h

## Usage

```
synchrotron_1(x, give=FALSE, strict=TRUE)
synchrotron_2(x, give=FALSE, strict=TRUE)
```

## Arguments

| | |
|---|---|
| x | input: real values |
| give | Boolean with TRUE meaning to return a list of three items: the value, an estimate of the error, and a status number |
| strict | Boolean, with TRUE meaning to return NaN if status is an error |

## Author(s)

Robin K. S. Hankin

## Examples

```
x <- seq(from=0,to=2,by=0.01)
synchrotron_1(x)
synchrotron_2(x)
```

---

# Transport *Transport functions*

---

## Description

Transport functions as per the Gnu Scientific Library, reference manual section 7.29. These functions are defined in header file gsl_sf_transport.h

## Usage

```
transport_2(x, give=FALSE, strict=TRUE)
transport_3(x, give=FALSE, strict=TRUE)
transport_4(x, give=FALSE, strict=TRUE)
transport_5(x, give=FALSE, strict=TRUE)
```

## Arguments

| | |
|---|---|
| x | input: real values |
| give | Boolean with TRUE meaning to return a list of three items: the value, an estimate of the error, and a status number. |
| strict | Boolean, with TRUE meaning to return NaN if status is an error. |

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
x <- seq(from=0,to=2,by=0.01)
transport_2(x)
transport_3(x)
```

---

Trig *Trig functions*

---

## Description

Trig functions as per the Gnu Scientific Library, reference manual section 7.30. These functions are declared in header file `gsl_sf_trig.h`

## Usage

```
gsl_sf_sin(x, give=FALSE, strict=TRUE)
gsl_sf_cos(x, give=FALSE, strict=TRUE)
hypot(x, y, give=FALSE, strict=TRUE)
sinc(x, give=FALSE, strict=TRUE)
complex_sin(zr, zi=NULL, r.and.i=TRUE, give=FALSE, strict=TRUE)
complex_cos(zr, zi=NULL, r.and.i=TRUE, give=FALSE, strict=TRUE)
lnsinh(x, give=FALSE, strict=TRUE)
lncosh(x, give=FALSE, strict=TRUE)
```

## Arguments

| | |
|---|---|
| `x,y` | input: real values |
| `zr` | In `gamma_complex()`, the real part of the argument |
| `zi` | In `complex_sin()` et seq, the imaginary part of the argument. If missing (ie takes the default value of `NULL`), interpret `zr` as complex, even if real |
| `r.and.i` | In `complex_sin()` et seq, Boolean variable with default value of `TRUE` meaning to return a complex variable as per the details section below; and `FALSE` meaning to return the values as advertised in the GSL manual |
| `give` | Boolean with `TRUE` meaning to return a list of three items: the value, an estimate of the error, and a status number |
| `strict` | Boolean, with `TRUE` meaning to return `NaN` if status is an error |

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
x <- seq(from=0,to=2,by=0.01)
gsl_sf_sin(x)   #table xx of Ab and St
gsl_sf_cos(x)   #table xx of Ab and St

f <- function(x){abs(sin(x+1)-sin(x)*cos(1)-cos(x)*sin(1))}
g <-
function(x){abs(gsl_sf_sin(x+1)-gsl_sf_sin(x)*gsl_sf_cos(1)-gsl_sf_cos(x)*gsl_sf_sin(1))}

f(100000:100010)
g(100000:100010)
```

---

# Zeta *Zeta functions*

---

## Description

Zeta functions as per the Gnu Scientific Library 7.31 and AMS-55, section 23.2. These functions are declared in header file `gsl_sf_zeta.h`

## Usage

```
zeta_int(n, give=FALSE, strict=TRUE)
zeta(s, give=FALSE, strict=TRUE)
zetam1_int(n, give=FALSE, strict=TRUE)
zetam1(s, give=FALSE, strict=TRUE)
hzeta(s, q, give=FALSE, strict=TRUE)
eta_int(n, give=FALSE, strict=TRUE)
eta(s, give=FALSE, strict=TRUE)
```

## Arguments

| | |
|---|---|
| `n` | input: integer values |
| `s,q` | input: real values |
| `give` | Boolean with `TRUE` meaning to return a list of three items: the value, an estimate of the error, and a status number. |
| `strict` | Boolean, with `TRUE` meaning to return `NaN` if status is an error. |

## Author(s)

Robin K. S. Hankin

## References

https://www.gnu.org/software/gsl/

## Examples

```
n <- 1:10
cbind(n,zeta(n),eta(n))   #table 23.3, p 811


zeta_int(1:5)
zeta(c(pi,pi*2))
zetam1_int(1:5)
zetam1(c(pi,pi*2))
hzeta(1.1,1.2)
eta_int(1:5)
eta(c(pi,pi*2))
```

# Index